

Generator & motor operations

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1 Introduction

Broadly electrical machines can be classified as *Static* and *Rotating* types. While *Transformer* is an example of static machine, *D.C* , *Induction* and *Synchronous* machines are the examples of rotating machines. Now before going formally to rotating machines , we shall consider the following two *classical* examples in order to introduce the concepts of *generator* and *motor* mode of operation of electrical machines based on a single conductor moving in magnetic field.

1.1 Generator Mode

Consider a straight conductor of *active length* (the length which is under the influence of the magnetic field) l meter is placed over two friction less parallel rails as shown in the figure 1. the conductor is moving with a constant velocity v meter/second from left to right in the horizontal plane. In the presence of a vertical magnetic field directed from top to bottom of strength $B \text{ Wb/m}^2$, a voltage $e = Blv$ will be induced across the ends of the moving conductor. The magnitude of the voltage will be constant and the polarity will be as shown in the figure 1(b). In other words the moving conductor has become a seat of emf and one can replace it by battery symbol with an emf value equal to Blv Volts.

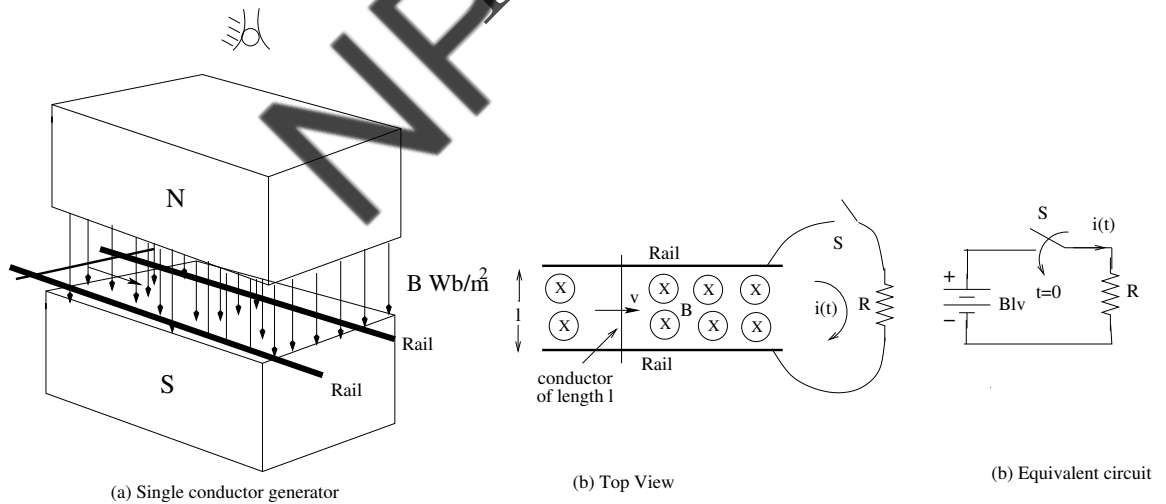


Figure 1: Single conductor generator.

As no load i.e., (resistance in this case) is connected across the moving conductor, output current hence output power is zero. Input power to the generator should also be zero which can also be substantiated by the fact that no external force is necessary to move a mass with constant velocity over a frictionless surface. The generator is said to be under no load condition. Let us now examine what is going to happen if a resistance is connected across the source. Obviously the conductor starts delivering a current $i = \frac{e}{R}$ the moment resistance is connected. However we know that a current carrying conductor placed in a magnetic field experiences a force the direction of which is decided by the *left hand rule*. After applying this rule one can easily see that the the direction of this electromagnetic force will be opposite to the direction of motion i.e., v . As told earlier that to move

the conductor at constant velocity, no external force hence prime mover is not necessary. under this situation let us assume that a load resistance R is connected across the conductor. Without doing any mathematics we can purely from physical reasoning can predict the outcome. The moment load is connected, the conductor starts experiencing a electromechanical force in the opposite direction of the motion. Naturally conductor starts decelerating and eventually comes to a stop. The amount of energy dissipated in the load must have come from the kinetic energy stored in the conductor.

Let us now Analyse the above phenomena mathematically. Suppose,

v_0 = linear velocity of the conductor in meter/sec under no load condition.

$t = 0$, is the instant when the load is switched on.

v = linear velocity of the conductor in meter/sec at any time t .

l = length of the conductor in meters.

m = mass of the conductor in Kg.

B = flux density in Wb/meter sq.

e = Blv , induced voltage at any time, t .

R = load resistance in Ω .

i = $\frac{e}{R}$, current in A at any time, t .

F_e = Bil , electromagnetic force in opposite direction of motion, at time, t . (1)

The dynamic equation of motion of the conductor can be written by using Newton's law of motion as follows:

$$\begin{aligned} m \frac{dv}{dt} &= -F_e = -Bil = -Bl \left(\frac{Blv}{R} \right) \\ m \frac{dv}{dt} + \frac{B^2 l^2}{R} v &= 0 \\ \frac{dv}{dt} + \frac{B^2 l^2}{mR} v &= 0 \text{ dividing both sides by } m. \end{aligned} \quad (2)$$

Solving this linear simple first order differential equation and applying the boundary condition that at $t = 0$, $v = v_0$ we get the expressions for velocity, emf and current as a function of time.

$$\begin{aligned} v &= v_0 e^{-\frac{B^2 l^2}{mR} t} \\ e &= Blv_0 e^{-\frac{B^2 l^2}{mR} t} \\ i &= \frac{Blv_0}{R} e^{-\frac{B^2 l^2}{mR} t} \end{aligned} \quad (3)$$

From the above we see that in absence of any external agency for motive power, the velocity and current decreases exponentially with a time constant $\tau = \frac{mR}{B^2 l^2}$ down to zero, as shown in the figure 2. We can easily calculate the amount of energy W_R dissipated in R and show the same to be equal to the initial kinetic energy ($\frac{1}{2}mv_0^2$) by carrying out the following integration.

$$W_R = \int_0^\infty i^2 R dt = \frac{1}{2}mv_0^2 \quad (4)$$

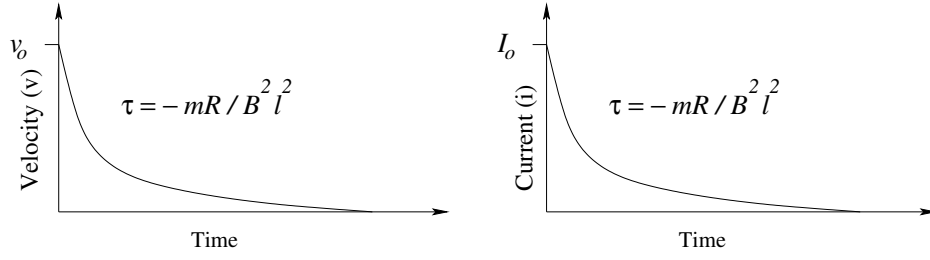


Figure 2: Velocity and Current variation

1.2 Motor Mode

consider the arrangement as shown in figure 3, Suppose the conductor is initially stationary, and battery of emf E is connected across it through a resistance R . Obviously the current right at the time of connecting the battery i.e., at $t = 0$, is $I = \frac{E}{R}$. As the current carrying conductor is placed in a magnetic field it will experience a force Bil . and the conductor will start moving towards right. However when the conductor starts moving a voltage is induced across the conductor the polarity of which will be such so as to oppose the inflow of current into the conductor. Therefore with time, both the values of the current and the electromagnetic force will decrease. To answer what will happen finally to the current and the speed of the conductor, we shall write the following electrodynamic equation and solve them.

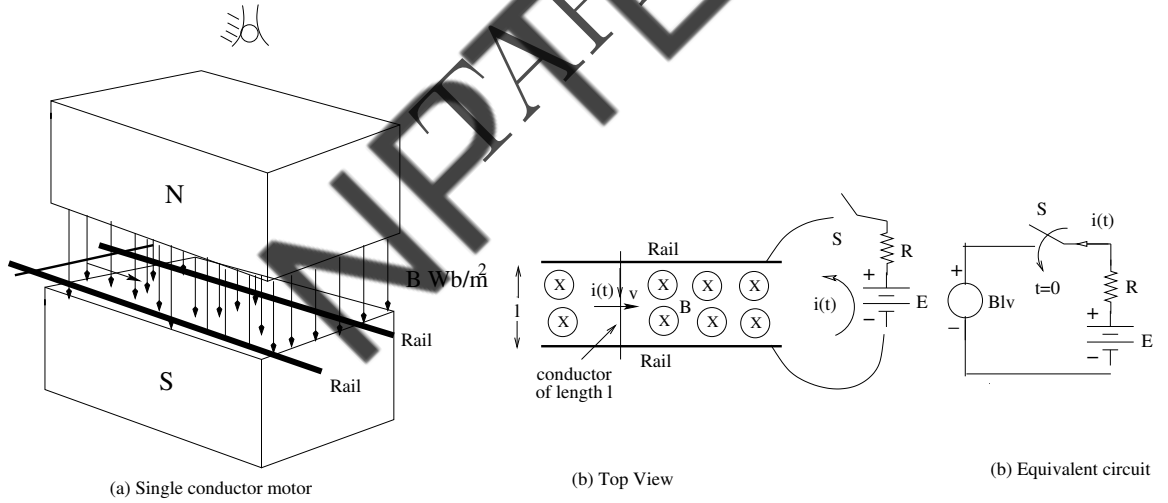


Figure 3: Single conductor motor.

- $t = 0$, is the instant when the battery is switched on.
- v = linear velocity of the conductor in meter/sec at any time t .
- l = length of the conductor in meters.
- m = mass of the conductor in Kg.
- B = flux density in Wb/meter sq.
- $e = Blv$, induced voltage at any time, t .
- R = resistance connected in series with the battery Ω .
- $i = \frac{E - e}{R}$, current in A at any time, t .
- $F_e = Bil$, electromagnetic force causing motion, at time, t .

The equations of motion in case of generator mode are as follows:

$$\begin{aligned}
 m \frac{dv}{dt} &= F_e = Bil = Bl \left(\frac{E - Blv}{R} \right) \\
 m \frac{dv}{dt} + \frac{B^2 l^2}{R} v &= \frac{BlE}{R} \\
 \frac{dv}{dt} + \frac{B^2 l^2}{mR} v &= \frac{BlE}{mR} \quad \text{dividing both sides by } m.
 \end{aligned} \tag{6}$$

Solving the above equation with the boundary condition, at $t = 0$, $v = 0$, the expressions for velocity and current are obtained as follows:

$$\begin{aligned}
 v &= \frac{E}{Bl} \left(1 - e^{-\frac{B^2 l^2}{mR} t} \right) \\
 e = Blv &= E \left(1 - e^{-\frac{B^2 l^2}{mR} t} \right) \\
 i = \frac{E - e}{R} &= \frac{E}{R} e^{-\frac{B^2 l^2}{mR} t}
 \end{aligned} \tag{7}$$

The figures 4 and 5 show the variation of current, velocity and emf induced in the conductor under motor mode condition.

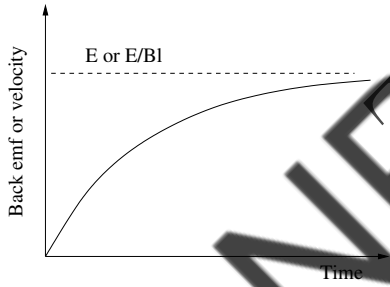


Figure 4: Variation of Velocity or emf.

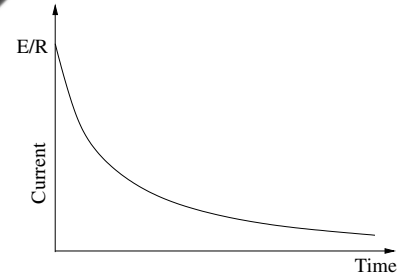


Figure 5: variation of current.

The readers can try to deduce how the above generator and motor will behave under loaded conditions.